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AN ELEMENTARY POLITICAL AND ECONOMIC THEORY OF THE EXPENDITURES OF LOCAL GOVERNMENTS*

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I. INTRODUCTION

It seems clear that problems in the field of public finance contain both political and economic elements. Yet, political scientists do not seem to have devoted a major part of their research efforts to the area, and economists traditionally have overlooked the political aspects of the problems.[1] This paper proposes a particular "political" theory of the expenditures of local governments with the aid of some of the traditional tools of economic analysis, and examines some data referring to the governments of the counties of Pennsylvania in light of the proposed theory.

It should be admitted at the outset, however, that the model developed herein is overly simple and, perhaps, naive. Yet, the authors believe that it has explanatory power, despite the fact that it requires the usual economic assumption of full knowledge, and that it represents a step in a desirable direction. Although the conceptual possibility of subjecting the model to a "direct test" is clearly evident, available data do not permit such a test and a sympathetic interpretation of the empirical results requires the admission of additional and rather strict assumptions. Hence, the empirical results do not constitute a convincing test of the major implication but merely serve to indicate that this theory, even when augmented with additional assumptions, seems to add explanatory potential to the standard models.

II. THE POLITICAL APPROACH TO PUBLIC FINANCE

This particular "political approach" has its intellectual basis in the works of Downs [7], Buchanan and Tullock [2], and Tullock [13]. Accordingly, it uses a particular view of the political process which is certainly not the only possible view, but which simply seems to be useful for the problem at hand. The basic postulate of this view is that in a democratic society individual voters are the underlying determinants of political decisions. Politicians, who actually make both expenditure and taxation decisions, are conceived as being motivated mainly by a desire to attain and remain in power (office).

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[1] Of course, Wicksell (for example, see [15)] is a major exception.

Hence, politicians, or at least successful ones, must both promise and actually make decisions which result in a "mix" of expenditures and taxes which will appeal to a dominant coalition of voters.

Clearly, in this particular view politicians represent participants in a sort of "guessing game." There exists a set of issues—e.g., the levels of various types of expenditures and the rates of the different taxes—which must be resolved. Aspirants and incumbents compete via the electoral process by taking stands on these issues. Each voter compares the positions taken by each candidate and votes for that politician whose stand most clearly corresponds to the one which he prefers. Hence, the winning politician is the one who makes the best approximation to the expenditure-taxation mix which can cause the formation of a dominant coalition of voters.

Individual voters, on the other hand, are assumed to be motivated by self-interest. They desire those expenditures and taxes which would maximize their utility, and they favor that candidate whom they anticipate will make those decisions most resembling the ones considered by them to be optimal.

Prior to the construction of a formal model, it is appropriate to discuss briefly some of the constraints upon the strategies which politicians can use and policies which they can espouse in their effort to get elected. Just any strategy will not do. For example, one might suggest that any politician who hoped to win should promise to tax only a specified minority and exclude that minority from the benefits of public expenditure while a specified majority received all such benefits but was not required to pay taxes. This type of strategy cannot be admitted. One reason may be that informal, ethical constraints exist in the real world which prevent the arbitrary designation of certain persons as being exempt from taxes. More importantly, there exist legal and constitutional restrictions upon the utilization of governmental powers. Politicians cannot arbitrarily designate certain individuals to be the sole beneficiaries of public expenditures and neither can they discriminatorily decide who is to be taxed and who is not. In the case of local governments, state laws generally designate the allowable areas of expenditure and types of taxes. Granted this framework, politicians can promise (at election time) or choose (if in office) areas and levels of expenditure and types and rates of taxes. Hence, strategies are constrained and winning majorities can be benefited relative to losing minorities only as expenditures and taxes fall unequally upon the members of the population. As a first approximation, attention is centered upon only these variables—the levels of expenditures and the tax rate—and their "impact" upon various groups in the population is considered in attempting to construct a political theory of local finance.

III. THE SIMPLE CASE OF ONE EXPENDITURE AND A PROPERTY TAX

Consider the following definitions:

- N: The number of persons registered to vote in a locality. (The term "locality" is used to mean the area under the jurisdiction of a local government.)
- x: The expenditure of the local government.
- q_{ik}: The quantity of the k^{th} private good—that is, a good which is purchased in the private sector of the economy—which the i^{th} individual purchases and consumes during the period under consideration.
- p_k: The price of the k^{th} good.
- f_i: The utility function of the i^{th} individual. This function is assumed to be both differentiable and strictly concave (downwards). Its arguments are private goods and the expenditure of the local government.
- I_i: The income of the i^{th} individual for the period under consideration.
- P_i: The assessed value of the taxable property owned by the i^{th} individual.
- r: The tax rate on the assessed value of property.[2]

It is appropriate, before proceeding, to

[2] It will be evident that the model below is easily adopted to other types of taxation. However, a property tax is most "realistic" from the point of view of local governments.

comment briefly upon the implicit assumptions inherent in the definition of the utility functions, the f_i. Admittedly, the introduction of public goods causes something of a problem for the economic theory of consumer behavior.[3] It is often the case that there is no "natural" unit with which to measure the quantity of a public good which an individual consumes—e.g., supposedly all individuals consume "general government," but it is difficult to determine how much is assigned to any one individual. The imperfect procedure adopted here simply is to presume that any utility function has as its arguments private goods and the (by assumption only one) expenditure of the local government. While it may be true that voters are concerned only with that portion of governmental expenditures which provide those goods and services which they consider themselves to be consuming, it appears appropriate to abstract from problems related to the manner in which governmental goods and services are distributed (and also produced). It is presumed that the f_i measure, as a function of x, the utility which individual voters assign to their consumptions of governmental goods and services. Presumably, this assumption can be taken to mean that both production efficiencies and distributional patterns are given and known so that they may be thought of as being incorporated into the f_i. Also, it is obvious that the absence of either inflation or a deflation must be assumed in order to avoid complicated alterations in the definition of the f_i.

Let the State Constitution require that the budget be balanced annually. Then assuming that the only source of revenue for the local government is the property tax, it follows that

$$(1) \qquad x = \sum_{j=1}^{z} rP_j$$

so that if a politician makes a choice of the value of x, then at the same time he implicitly makes a choice of r.

Assume for the moment that any given voter-consumer considers himself free to select the value of x. Then, assuming that the period under consideration is the same as that over which the budget is balanced, a model of individual behavior may be written as follows:

$$(2.1) \qquad \max f_i(q_{i1}, \cdots, q_{is}, x)$$

(2) subject to

$$(2.2) \qquad \sum_{k=1}^{s} p_k q_{ik} + rP_i = I_i$$

where (2.1) represents the i^{th} voter-consumer's utility function and (2.2) is his budget constraint which states that his annual income must be disposed of by paying taxes and spending on goods (unless one of the q_{ik} is interpreted as "saving"). In order to state the budget constraint (2.2) in a more desirable form, use (1) in substituting for r to obtain

$$(3) \qquad \sum_{k=1}^{s} p_k q_{ik} + \frac{x}{\Sigma P_j} P_i = I_i$$

as an alternative form of the budget constraint.

Consider now the maximization of (2.1) subject to the constraint (3). Using Lagrangian methods, one obtains

$$(4.1) \quad \frac{\partial f_i}{\partial q_{ik}} - \lambda_i p_k = 0, \quad k = 1, \cdots, s$$

as the conditions for a consumer maximum in the market for private goods. Note that λ_i, the multiplier associated with the budget constraint (3), can be interpreted as the i^{th} consumer's marginal utility of income. Assuming $\lambda_i > 0$ and defining $\alpha_i = 1/\lambda_i$, let (4.1) be written in the form

$$(4.2) \quad \alpha_i \frac{\partial f_i}{\partial q_{ik}} - p_k = 0, \quad k = 1, \cdots, s$$

which is the same result as is obtained from the traditional economic model of consumer behavior. The consumer must select that

[3] It should be emphasized that the "public goods" under consideration here need not be the polar type defined by Samuelson [10].

quantity of the k^{th} good which will equate price and the weighted marginal utility of that good.

Of more interest here is the optimality condition for the local expenditure:

$$(5.1) \qquad \frac{\partial f_i}{\partial x} - \lambda_i \frac{P_i}{\Sigma P_j} = 0$$

and upon dividing by λ_i and substituting as above one has

$$(5.2) \qquad \alpha_i \frac{\partial f_i}{\partial x} - \frac{P_i}{\Sigma P_j} = 0$$

as a more interpretable form of this condition. This result is basic to the argument.

Consider carefully the meaning of (5.2). If the i^{th} voter-consumer is not a property owner ($P_i = 0$), then he will view the municipal expenditure as a "free good" and will desire that value of x to be selected which equates his marginal utility of the expenditure to zero.[4] On the other hand, if the i^{th} voter-consumer is a property owner

$$(P_i > 0),$$

then he will want that value of x to be selected which equates his marginal utility of the expenditure, weighed by the reciprocal α_i of his marginal utility of income λ_i, to the ratio of the value of his property over the value of all taxable property in the locality. Assume that the public expenditure is not an inferior good for anyone. Then note that this interpretation implies that if two individuals have identical tastes and incomes, but one owns property and the other does not, then, under the usual assumption of diminishing marginal utility, the property owner will desire a smaller local expenditure than will the non-property owner.[5] The ratio $P_i/\Sigma P_j$ fills the role usually assigned to prices in the conventional theory of consumer behavior. Furthermore, with a given f_i and with I_i held fixed, if the parameter ΣP_j is increased in value in such a manner that $P_i > 0$ is held constant, then, assuming that the public expenditure is not inferior, the i^{th} voter-consumer will desire the value of x to be increased in order to maintain equality in (5.2).[6] On the other hand, if ΣP_j is increased without $P_i > 0$ being held constant, then, even with the assumption that the public expenditure is not inferior, the desired change in the value of x will depend upon whether the ratio $P_i/\Sigma P_j$ increases or decreases.[7]

[4] Although the P_i are assessed values and do not depend upon r, it should be observed that this argument depends upon the presumption that property taxes are not shifted to renters; although the qualitative (or slightly modified) conclusion remains valid as long as the tax is not fully shifted as can easily be seen by putting the additional term representing the marginal tax shift into (5.2). Nevertheless, the following argument explains why it appears appropriate to conduct the analysis on the basis of the strict assumption that the property tax is not shifted. Imagine a supply curve of homogeneous rental properties. In the short run this supply is viewed best as a fixed stock (vertical curve) since additions or deletions are a negligible portion of the whole. In any event, the imposition of a tax should not cause withdrawals from the supply since such an action would not diminish the tax liability; and the supply can be viewed as being fixed. The demand for rental properties does not depend upon the tax. Hence, the property tax will not be shifted in the short run. In addition, the long run is indeed long in terms of tax shifting since sufficient time must be allowed for the tax to prevent what would have otherwise been a non-negligible addition to the stock of rental properties.

[5] Obviously, one must assume here that the local expenditure is one which is desired by both individuals and not one which, say, merely enhances the value of the property owner's property.

[6] It is apparent that the phenomenon described above is analogous to the "substitution effect" in the traditional theory of consumer behavior. Obviously, there is an associated "income effect" which makes it necessary to qualify the above statement with the assumption that the public expenditure is not inferior.

[7] It is apparent that, in certain instances, the assumption that governmental expenditure equals tax revenue can be relaxed without substantially altering the above results. Suppose, for example, that the locality obtains a subsidy of C dollars from the State. Then by definition

$$(a) \qquad x = \Sigma\, rP_j + C$$

so that upon solving for r, substituting the result into (2.2), and deriving the conditions for a maximum, one finds that the new condition concerning the municipal expenditure is of the same form as (5.2). On the other hand, if the State grants a

Up to this point it is assumed that each individual feels free to select the value of x. In fact, the value of x is politically determined so that it remains to incorporate the above developments into a larger model. In doing this, define the value of x (denoted $\bar{x}_i$) which the i^{th} individual would select if free to do so to be the i^{th} voter-consumer's "preferred point." This concept plays an important part in the discussion which follows.

Since different individuals are likely to have different preferred points (for many reasons), and since the political process can select only one value of x for a given period of time, it is to be expected that some individuals in the populace will experience utility "losses" due to the fact that the politically determined governmental expenditure is not identical to the desired expenditure (or preferred point). In order to formalize this notion of losses due to deviations from the optimum, define $\bar{q}_{ik}$ to be the quantity of the k^{th} good chosen by the i^{th} voter-consumer when $x = \bar{x}_i$ is also selected. In other words, the $\bar{q}_{ik}$, $(k = 1, \cdots, s)$, and $\bar{x}_i$ are the quantities which constitute the optimal solution to (2). Next, note that if x is regarded as a variable whose value cannot be selected by the i^{th} consumer, then for any given value of x (and under assumption (1) this means that a value of r is also specified), problem (2) can be solved for optimal values of the q_{ik}. Let q_{ik}', $(k = 1, \cdots, s)$, represent these optimal values for any given values of x. Note that with P_i, I_i, and the p_k being parameters, the q_{ik}' are functions of x. Define

$$(6) \quad F_i(x) = f_i(\bar{q}_{i1}, \cdots, \bar{q}_{is}, \bar{x}_i) - f_i(q_{i1}', \cdots, q_{is}', x)$$

to be the i^{th} voter-consumer's "loss function." Note that, unlike ordinal utility functions, the loss function has a "natural" zero point at $F_i(\bar{x}_i) = 0$, the i^{th} voter-consumer's preferred point.[8] The loss function (6) indicates the utility losses experienced by the i^{th} individual when x assumes any assigned value. A typical loss function is shown in Figure 1. Note especially that since the f_i are strictly concave (downwards), then

$$d^2F_i/dx^2 > 0$$

and the F_i are strictly convex (concave upwards). Thus $F_i(x) \geq 0$ for all x and $F_i(x) = 0$ if and only if $x = \bar{x}_i$.

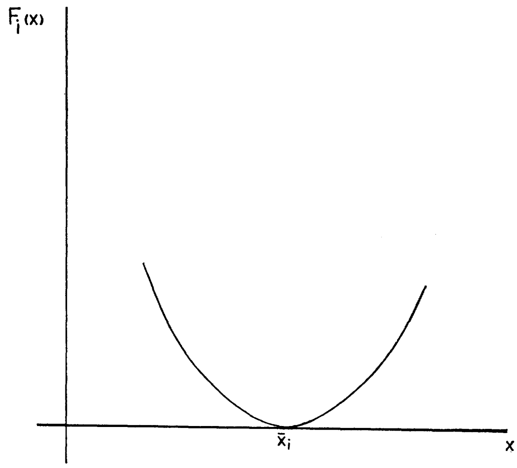

FIGURE 1

It is now apparent that the electorial-decision process of the political phenomenon can be linked to voter-consumer desires. Suppose that any locality has two candidates (or slates of candidates put up by the political parties or factions) and that these can-

subsidy of a certain percentage c of the municipal expenditure, then one has

$$\text{(b)} \qquad x = \Sigma\, rP_j + cx$$

so that upon solving for r, substituting the result into (2.2), and making the appropriate derivations, one finds that

$$\text{(c)} \qquad \alpha_i \frac{\partial f_i}{\partial x} - \frac{(1-c)P_i}{\Sigma P_j} = 0$$

is the new condition concerning the municipal expenditure. In both of these instances the subsidy may alter the value of λ_i.

[8] The notion of individual loss functions, while familiar in statistical decision theory, is introduced and used in the context of committee decisions in van den Bogaard and Versluis [14] and Theil [11, 12].

didates are denoted Star (*) and Prime ($'$) respectively. The winning politician is presumed to be the candidate receiving a simple majority of the votes. Before the election day, each candidate is assumed to announce his platform, which consists solely of the value of x which he intends to select if elected. All registered voter-consumers are assumed to know the platform of each of the candidates, and all registered voters cast a ballot on election day. Further, in order to rule out ambiguity it is assumed not only that the winning politician has the power to select, but also that he in fact does select, the value of x specified in his platform.[9]

Let x^* represent the platform of the candidate Star, and let x' represent Prime's platform. Each voter-consumer chooses between these two candidates on the basis of these platforms, and the basic motivational assumption is that the voter-consumers desire to maximize their individual utilities or, equivalently, minimize their utility losses as these are given by (6). In terms of the i^{th} voter-consumer's utility losses, the voting rules can be stated as follows:
If $F_i(x^*) < F_i(x')$, vote for Star. If

$$F_i(x^*) > F_i(x'),$$

vote for Prime. If $F_i(x^*) = F_i(x')$, toss a fair coin in order to make the decision.

Observe that the preferred points $\bar{x}_i$, $(i = 1, \cdots, N)$, can be arranged into a frequency according to the values of the $\bar{x}_i$. Let $\bar{x}_m$ denote the (not necessarily unique) median of the frequency of the $\bar{x}_i$. It is now possible to establish the following theorem:

Theorem 1: Given the assumed characteristics of the individual loss functions (6), the rules for voters' choices, and the fact that exactly two candidates compete in the election, then a platform $\bar{x}_m$ is dominant in the sense that no non-median platform exists which will give at least one-half of the voters a smaller utility loss (or larger utility gain) than will a platform $\bar{x}_m$.

Proof: Let the candidate Prime choose the platform $x' = \bar{x}_m$ and the candidate Star select a non-median platform satisfying $x^* > \bar{x}_m$. By the definition of the median, there are at least $N/2$ voters whose preferred points satisfy the relationship $\bar{x}_i \leq \bar{x}_m$. Let e denote some voter-consumer selected arbitrarily from the $N/2$ individuals for which this relationship $(\bar{x}_i \leq \bar{x}_m)$ obtains. However, all loss functions are strictly convex (concave upwards) and $F_e(\bar{x}_e) = 0$ where the relevant points can be ordered

$$\bar{x}_e \leq \bar{x}_m < x^*.$$

Therefore, $F_e(x') = F_e(\bar{x}_m) < F_e(x^*)$. It is obvious that if the candidate Star selects a non-median platform $x^* < \bar{x}_m$, then this argument can be reversed. QED.

Given the voting rules, Theorem 1 means that if one candidate selects the median of the desired expenditures (preferred points) to be his platform, and the other candidate selects some non-median strategy, then that candidate selecting the median is certain to win the election. Of course, if both candidates choose a median strategy, then a tie is expected since the election is equivalent to a toss of a coin. However, the dominance of the median should be a powerful inducement for politicians to select such a platform, for the choice of a non-median strategy only invites defeat. Further, if the election is viewed as a two-person zero-sum game, then the choice of the median preferred point is the strategy prescribed by the famous minimax theorem.[10]

It may be true, of course, that politicians do not select their strategies according to the minimax theorem. Nor do the authors argue that minimax is "rational behavior." [11]

[9] This assumption is not as farfetched as it might seem if it is assumed that the values of the parameters remain fixed during the period in which the winning politician is in office. For an interesting argument concerning the "validity" of this assumption, see Downs [7], pp. 103–109.

[10] For a statement of the minimax theorem and a discussion of two-person zero-sum games, see chapter 4 of Luce and Raiffa [9].

[11] For an interesting discussion of this point, see Luce and Raiffa [9], pp. 62–63.

It seems obvious that the choice of an electoral strategy depends, at least in part, upon the evaluation of the actual or expected strategy of the opposing candidate. Thus, if the opposition happens, for some reason, to take an extreme position; a candidate might desire to take advantage of the situation and select a certain non-median strategy in order to roll up an impressive majority which might enhance his chances of moving to a higher elective office in the system at some later date.[12] However, it may be suggested that since non-median strategies invite defeat if the opposition is shrewd, those candidates who can win in election after election are likely to be the ones who make their choices, at least approximately, by the minimax theorem.[13] Thus it is argued that there should be a "tendency" for winning politicians to be the ones who attempt to find and select the median of the frequency of preferred points.

IV. ON THE EXPLANATORY POWER OF A SIMPLE CASE

There are several reasons why the theory developed in the previous section is not in a form particularly suitable for empirical testing. Probably the most important of these reasons is that Theorem 1 refers to a median $\bar{x}_m$ of the frequency of preferred points. This median (and, indeed, the entire frequency) is non-observable. While one can imagine politicians attempting to estimate (explicitly or implicitly) a median, the independent estimation (by the authors) of medians of the preferred points of registered voters in various localities appears to be a prohibitively expensive method of testing this theory. Yet, even a crude attempt toward empirical justification seems important.

The empirical strategy used involves much stronger assumptions than those required for the development of the theory. The additional assumptions which are to be made here are admittedly unrealistic. However, at least from one point of view, models with descriptively unrealistic assumptions often provide valuable insight into the complexities of the real world and help to determine the relative importance of various theoretical presumptions. Accordingly, some evidence is examined in an effort to determine whether this model, with the addition of some unrealistic assumptions, has any explanatory power.

The particular evidence which is examined refers to the county governments of Pennsylvania. Each county is governed by three County Commissioners who are charged with the responsibility of determining expenditures and taxes. The commissioners are selected under a system where each registered citizen has two votes and the three candidates with the highest number of votes are the winners. Since in practice each party nominates two candidates who tend to adopt approximately the same platform, and any two commissioners form a majority, it is presumed that the majority commissioners are equivalent to a winning candidate so that this aspect of the model roughly approximates reality. Furthermore, Pennsylvania's Act 481—the so-called "Tax Anything Law"—has never been extended to the counties so that for practical purposes these governments are dependent upon the property tax. Hence, it is presumed that this model applies to these data.[14]

[12] Professor Julius Margolis points out that the objectives of local politicians might be to tie up various factions of the electorate and "win big" in order to move up the political ladder to, say, the state level. Margolis argues further that minimax strategies might lead to incumbency but not to advancement.

[13] It should be pointed out that, if the opposition is not shrewd, minimax strategies can still lead to "winning big." Also, merely winning is almost a necessary condition in many systems for political advancement so that defeats are to be avoided at all cost. The previous theorem of this paper indicates that the minimax strategy of selecting the median is the best way of avoiding defeats. Hence, even ambitious politicians may find it rational to select the median strategy.

[14] Two of Pennsylvania's counties—Forest, for which complete data are not available, and Philadelphia, which has a joint city-county government—are omitted from the data.

In trying to use cross sectional data to determine the explanatory power of this model, several problems are evident. First, at this stage of development the model allows only one expenditure, and all counties have several expenditures. The approach used is to consider "the expenditure" to be nothing more than the sum of all current spending except capital outlay and interest payments.[15] In other words, at this point in the analysis no distinction is made between various types of expenditures. Second, although in the development of the model the term "expenditure" was used to mean "total expenditure" (which certainly is acceptable since the model can be applied to particular local governments), the use of cross sectional data from counties with varying populations poses something of a problem. An adjustment is needed; and the most appropriate one appears to be to correct for the varying populations by considering per capita expenditures. This adjustment causes no difficulty for the theory since the developments are easily made in terms of per capita expenditures if one merely views the population parameter as being incorporated into the coefficients of the utility functions. Hence, from the point of view of the cross sectional data, the theory can be regarded as if the results were stated in terms of per capita expenditures.

Finally, there is the problem of the additional (and descriptively unrealistic) assumptions. Let it be assumed that the tastes and incomes of all registered voters in all of the localities under consideration are identical. Specifically, the tastes of the voters and the ordinal indices of the utility functions are such that $f_i = f_j$ for $i \neq j$; and the incomes of these voter-consumers are the same so that $I_i = I_j$ for $i \neq j$. However, there may be differences in the ownership of taxable property so that $P_i \neq P_j$ for $i \neq j$ is allowed. It is clear that these assumptions are such that the preferred points (in per capita terms) of voter-consumers can differ only because they own taxable properties of different assessed values; and it is irrelevant whether these voters reside in the same or different counties. In other words, if n_w and n_v represent the populations of the w^{th} and v^{th} counties respectively (and it does not matter whether $w = v$ or $w \neq v$), then $\bar{x}_i/n_w \neq \bar{x}_j/n_v$ for $i \neq j$ if and only if their respective property ratios are unequal—that is, only if $P_i/\Sigma P_k \neq P_j/\Sigma P_t$.

The above assumptions have at least two important implications. First, within any locality (or county) there is a one to one correspondence between the preferred points $\bar{x}_i$ (and equivalently, the preferred points in per capita terms $\bar{x}_i/n$ where n is the population of the county) and the property ratios $P_i/\Sigma P_j$ of the voter-consumers in that locality. On the basis of the assumption that the public expenditure is not inferior, this correspondence defines an inverse relationship—the larger the ratio $P_i/\Sigma P_j$ the smaller the desired per capita expenditure $\bar{x}_i/n$. Further, a median preferred point $\bar{x}_m$ (or equivalently, $\bar{x}_m/n$) is associated with a corresponding median property ratio $P_m/\Sigma P_j$. Politicians can determine a median preferred point by finding a median property ratio. Second, if politicians do tend to follow the minimax strategy defined by Theorem 1, and if the additional assumptions are also satisfied, then in the cross section of Pennsylvania counties one should observe an inverse relationship between the actual per capita expenditures of these counties and the corresponding median property ratios. Thus, the presence (or absence) of this relationship should indicate whether the model (with the additional assumptions) has explanatory

[15] These categories are excluded because of the following reasons: capital outlay tends to be "lumpy" over time, occurring during some period when a new facility is constructed and then is not required again until some other new facility is desired. Thus current capital expenditure is a function of past capital expenditure and, perhaps, certain other variables. Similarly, interest payments are a function of past commitments and depend upon such things as the time preferences of voters, financing schemes, the interest rate prevailing at the time of borrowing, etc.

power. Unfortunately, it is also impossible to determine directly whether this relationship obtains since data on individual property holdings are not reported. An alternative and approximatory method which uses available data is required.

The procedure adopted here involves the use of two surrogates.

y_1 : The per capita value (assessed) of all taxable property in a county in thousands of dollars.

y_2 : The number of owner-occupied residences in a county divided by the number of registered voters in that county. Assuming that occupants of these residences are voters, this variable approximates the percentage of the electorate that owns property.

Consider the following propositions:

Proposition 1: Relatively high values of y_1 are associated with relatively low values of $P_m/\Sigma P_j$.

Proposition 2: Relatively large values of y_2 are associated with relatively large values of $P_m/\Sigma P_j$.

While these two propositions certainly are not logically true, it is argued that they are reasonable for the purpose at hand.

First, consider the former proposition. It appears plausible to believe that this proposition is empirically true for several reasons. As one goes from county to county, relatively high values of y_1 are likely to be observed in the more urbanized areas. In such counties it is likely that a larger portion of the populace are renters so that the left tail of the density of the $P_i/\Sigma P_j$ may be relatively larger for this reason alone. In addition, the greater values of y_1 are probably associated with those counties where commercial and industrial properties are a relatively important part of the tax base. At least part of these properties are owned by stockholders residing in other counties or states. Such individuals are not included in the density of the $P_i/\Sigma P_j$, although the value of the property is included in the denominator of the term. Hence, it appears that large values of y_1 are likely to be associated with relatively small values of the median ratio $P_m/\Sigma P_j$.

Consider the latter proposition. It would seem that the larger the percentage of the electorate owning their residences, the more likely it is that a home owner is the median voter-consumer and the smaller is the left tail of the density of the $P_i/\Sigma P_j$. This argument indicates the plausibility of the second proposition.

Assuming that both of these propositions are true for the densities under consideration here, let n represent the population of any given county and consider the following equation

$$x/n = \beta_0 + \beta_1 y_1 + \beta_2 y_2 + \epsilon \tag{7.1}$$

where ϵ is an error term. Note that (7.1) can be fitted to available data by the method of ordinary least squares. On the basis of the above propositions and theoretical developments, $\beta_1 > 0$ and $\beta_2 < 0$ are expected; but since it can be argued that these inferences are not valid for the coefficients of a multiple regression, the coefficients in the simple regression equations

$$\text{(7.2)} \qquad x/n = \beta_3 + \beta_4 y_1 + \delta_1$$

$$x/n = \beta_5 + \beta_6 y_2 + \delta_2$$

were also estimated. Note that $\beta_4 > 0$ and $\beta_6 < 0$ are anticipated.[16]

Table I presents the estimates of the coefficients and associated statistics.[17] Note that

[16] Of course, the usual problems associated with ratio estimation are also present here. See, e.g., Kuh and Meyer [8].

[17] Sources of data are as follows: population and the number of owner-occupied residences per county can be found in the *U. S. Census of Population: Social and Economic Characteristics*. The *Pennsylvania Statistical Abstract* (Harrisburg; Pa.: Department of Internal Affairs, 1960), is the source of the data on voter registration. Both county expenditures and the assessed value of taxable property are reported in *Commonwealth of Pennsylvania Local Government Financial Statistics, 1959* (Harrisburg, Pa.: Department of Internal Affairs). It should be pointed out that an apparent mistake was uncovered for expenditures on general government, and hence for the sum of

TABLE I
TOTAL EXPENDITURES PER CAPITA
Multiple Regression

Number of Observations.........64
Multiple R...................... .6528
F statistic........................22.6458

Variable	Coefficient	Standard Error	T statistic
Y_1	5.2639	.8473	6.2125
Y_2	−0.1962	.0592	−3.3115

Simple Regression

Variable	Y_1	Y_2
Coefficient	4.9286	−.1522
Standard Error	.9063	.0746
T statistic	5.4380	−2.0417
Simple r	.5683	−.2510
F statistic	29.5717	4.1686

the signs of the coefficients for both the multiple and simple regressions are as predicted and that these estimated coefficients are significantly different from zero (in the desired directions) at the .01 level. While these results are highly encouraging, it should be noted that the fits are not exceptional and the multiple regression (which gives the best fit as might be expected) explains only a little better than 42 percent of the variance.[18] Yet, granted the crudeness of these approximatory variables, it appears that these data support the notion that this model, even when restricted by the addition of admittedly unrealistic assumptions, does have explanatory power.

V. THE CASE OF MULTIPLE EXPENDITURES

Consider extending the previous developments to the case of an arbitrary number of governmental expenditures. Again, assume a requirement of an annually balanced budget so that by definition

$$(8) \qquad \sum_{\tau=1}^{u} x_t = \sum_{j=1}^{z} rP_j$$

and if a politician makes a choice of the values of all of the x_t, then he is at the same time making a choice of the value of r.

Consider the following consumer maximization problem:

$$(9.1) \quad \max f_i(q_{i1}, \cdots, q_{is}, x_1, \cdots, x_u)$$

(9) subject to

$$(9.2) \qquad \sum_{k=1}^{s} p_k q_{ik} + rP_i = I_i$$

By using (8) to substitute for r in (9.2), applying Lagrangian maximization methods, letting $\alpha_i = 1/\lambda_i$ where λ_i is the multiplier associated with (9.2), one arrives at the necessary conditions for the consumer maximum:

$$(10.1) \quad \alpha_i \frac{\partial f_i}{\partial q_{ik}} - p_k = 0, \quad k = 1, \cdots, s$$

$$(10.2) \quad \alpha_i \frac{\partial f_i}{\partial x_t} - \frac{P_i}{\Sigma P_j} = 0, \quad t = 1, \cdots, u$$

where (10.1), which refers to private goods, corresponds to (4.2), and (10.2), which refers to governmental expenditures, corresponds to (5.2). Accordingly, the interpretations given for (4.2) and (5.2) apply also to (10.1) and (10.2) respectively.

Let $\bar{x}_{it}$, $(t = 1, \cdots, u)$, and $\bar{q}_{ik}$, $(k = 1, \cdots, s)$, be those values of the x_t and q_{ik} respectively which are the optimal solution to (9). Then the $\bar{x}_{it}$ define the i^{th} voter-consumer's preferred point for the expenditures of the local government. Recalling that the values of the x_t are selected in the political process, note for arbitrarily specified values of the x_t and from (8) also a value of r, that (9) can be solved for optimal values of the q_{ik}. Again, let q_{ik}', $(k = 1, \cdots, s)$, represent the optimal values of the q_{ik}, for given values of the x_t. Note that the q_{ik}' are functions of the x_t since P_i, I_i, and the p_k are

expenditures, for Bucks County. The computations reported in Table I and Table II were made on the basis of the adjusted figures for Bucks County.

[18] An examination of the residuals indicates that one county, Allegheny, contributes a rather large portion of the unexplained variance. Since this County, whose boundaries include the City of Pittsburgh, has a government with a more "metropolitan flavor" than those of the other counties, this fact may be somewhat understandable.

parameters. It is now possible to define

$$F_i(x_1, \cdots, x_u) = f_i(\bar{q}_{i1}, \cdots, \bar{q}_{is}, \bar{x}_{i1}, \cdots, \bar{x}_{iu}) - f_i(q_{i1}', \cdots, q_{is}', x_1, \cdots, x_u) \tag{11}$$

to be the i^{th} voter-consumer's loss function. Again, observe that $F_i(x_1, \cdots, x_u) \geq 0$ for all values of the x_t and that

$$F_i(\bar{x}_{il}, \cdots, \bar{x}_{iu}) = 0.$$

Suppose that there are two candidates, Star and Prime, in the locality and that each registered voter goes to the polls at election time and casts a vote for one of these two candidates. Let $(x_1^*, \cdots, x_u^*)$ denote Star's platform and $(x_1', \cdots, x_u')$ represent Prime's platform.[19] Suppose that each will honor his platform if elected. Assume that all voters know the candidates' platforms. Let the motivational assumption be that voters desire to maximize the individual utility or, equivalently, to minimize the individual utility losses. Hence, the i^{th} voter-consumer will choose Star if

$$F_i(x_1^*, \cdots, x_u^*) < F_i(x_1', \cdots, x_u')$$

and he will cast his ballot for Prime if

$$F_i(x_1^*, \cdots, x_u^*) > F_i(x_1', \cdots, x_u').$$

In the event

$$F_i(x_1^*, \cdots, x_u^*) = F_i(x_1', \cdots, x_u')$$

it is assumed that the voter will be as likely to select one candidate as the other.

Granted the apparent similarities between the multiple and single expenditure cases, one might suspect that a dominant political strategy should exist. Such may be the case, but the authors have not been able to demonstrate this possibility and it may be ruled out, at least under certain conditions, by Arrow's impossibility theorem.[20] However, what can be shown (See Appendix I) is that if utility functions are concave (downward) and can be written in the form

$$f_i(q_{i1}, \cdots, q_{is}, x_1, \cdots, x_u) = g_i(q_{i1}, \cdots, q_{is}) + h(x_1, \cdots, x_u) \tag{12}$$

then a dominant strategy does exist in the sense that no other strategy gives a higher utility return (smaller utility loss) to at least one-half of the voters. Note that f_i is "separable" between private goods and public expenditures, and that h is not subscripted so that it is assumed to be the same for all voter-consumers. These are, admittedly, rather stringent assumptions; but a dominant strategy does exist under these conditions. Further, this dominant platform is a strategy of the median type in the sense that it represents the preferred point of a voter-consumer whose desired total public expenditure is the (not necessarily unique) median of the frequency of the desired total public expenditures of the voter-consumers. This dominant platform is defined carefully in Appendix I. Reference is made to Theorem 2 which is stated and proved there.

As Theorem 2 is stated, there is no necessary relationship between the median of the frequency of the desired total public expenditures and the median of the frequency of property ratios. However, it is quite obvious from (10.2) that the assumptions of identical tastes and incomes create such a relationship. In other words, if $f_i = f_j$ and $I_i = I_j$ for $i \neq j$, then a voter-consumer with a median property ratio also has a median desired total public expenditure. The preferred point of such a voter-consumer is a dominant strategy.

VI. SOME RELEVANT DATA

Having admitted multiple expenditures, it is now appropriate to examine the data from the counties of Pennsylvania in somewhat greater detail. However, not all categories of expenditures are examined here for two reasons. First, an obvious non-uniformity in reporting practices makes some

[19] Note from (8) that the selection of a platform implies the choice of a tax rate.

[20] See Arrow [1]. In quite a different context (that of policy formation) the existence of a dominant political strategy is demonstrated under certain conditions and other related issues are discussed in Davis and Hinich [6].

of the minor categories suspect. Second, it is of special interest to examine those categories which should give insight into the question of whether this model, augmented with the additional and unrealistic assumptions of identical tastes and equal incomes, has explanatory power. Specifically, the following categories are examined: general government, highways, judicial, and all other expenditures except capital outlay and interest payments.

General government was selected as a category because the State defines the higher elective offices and the salaries to be paid to these office holders, but allows local authority to determine (i) whether certain appointive jobs are to exist, (ii) the salaries of these jobholders, and (iii) whether certain minor elective offices can be combined or have to be held by different individuals. This category is an example of an instance in which the local electorate has some but not full authority. Highways is an interesting category because much of the funds are obtained, not by taxes levied by the commissioners, but by the State's "returning" to the counties ear-marked monies obtained from the gasoline tax. In this instance, local authority is subjected to State influence in an indirect manner. The judicial category is especially interesting for this purpose because, although the State defines certain clerical offices, a great deal of leeway is left to local authority in organizing the local judicial system. Similarly, other expenditures—which includes corrections, charities, hospitals and health, libraries, civil defense, airports, and miscellaneous expenditures—is a category where local authority is more dominant. Thus in examining these categories of expenditures one should be able to ascertain, at least in a rough manner, the effects of certain institutional phenomena which were not included in this theory.

Ignoring these institutional phenomena for a moment, note that if the assumptions of the multiple expenditure case are satisfied simultaneously for the counties in Pennsylvania, if the additional and unrealistic assumptions of identical tastes and equal incomes are admitted, if it is assumed that no category of expenditure is an "inferior good," and if the theory is stated in terms of per capita expenditures (which, as was noted earlier, causes no difficulty), then the difference between the counties in the per capita expenditures in each category should be explained by the differences in the median ratios $P_m/\Sigma P_j$ of each county. Further, for each category there should be an inverse relationship between the per capita expenditure and the median ratio. Since these ratios cannot be observed, the procedure used in the previous instance, including the two propositions, is adopted here. Accordingly, consider the following multiple regression (where n represents the population of any given county)

$$(13)\quad x_t/n = \beta_{0t} + \beta_{1t}y_1 + \beta_{2t}y_2 + \epsilon_t \qquad t = 1, \cdots, 4$$

as well as the simple regressions

$$(14)\quad \begin{aligned} x_t/n &= \beta_{3t} + \beta_{4t}y_1 + \delta_{1t}, \qquad t = 1, \cdots, 4 \\ x_t/n &= \beta_{5t} + \beta_{6t}y_2 + \delta_{2t}, \qquad t = 1, \cdots, 4 \end{aligned}$$

where ϵ_t, δ_{1t}, and δ_{2t} are error terms. Granted the two propositions and that the assumptions are satisfied, then $\beta_{1t} > 0$, $\beta_{2t} < 0$, $\beta_{4t} > 0$, and $\beta_{6t} < 0$ are expected for all t.

Consider the effects of the institutional phenomena. It seems clear on an a priori basis that if these phenomena have an effect, then they should distort the predicted relationships. Thus one expects to obtain somewhat better fits for judicial and other expenditures than for general government and highways.

The empirical results are summarized in Tables II, III, IV, and V. Note that the estimated coefficients have the anticipated signs in all instances. However, for both highways and other expenditures the estimated coefficients for y_2 in both the simple and multiple regressions are not significantly different from zero at the .05 level. In all other instances, the estimated coefficients are significantly different from zero (in the predicted directions) at the .01 level. Although none of the fits are exceptional, it is interesting to note that, as measured by the multiple correlation coefficients, the best fit is obtained for judicial expenditures and the poorest for highways. On the other hand, the fit for general government is almost as good as that for judicial expenditures, so that the antici-

TABLE II

PER CAPITA EXPENDITURES ON GENERAL GOVERNMENT

Multiple Regression

Number of Observations 64
Multiple R .... .5417
F statistic 12.6689

Variable	Coefficient	Standard Error	T statistic
Y_1	1.2241	.3024	4.0482
Y_2	−0.0731	.0211	−3.4539

Simple Regression

Variable	Y_1	Y_2
Coefficient	1.0993	−.0628
Standard Error	.3256	.0234
T Statistic	3.3762	−2.6777
Simple r	.3941	−.3220
F Statistic	11.3989	7.1702

TABLE III

PER CAPITA EXPENDITURES ON HIGHWAYS

Multiple Regression

Number of Observations 64
Multiple R .... .4002
F statistic 5.8155

Variable	Coefficient	Standard Error	T Statistic
Y_1	1.2057	.3691	3.2663
Y_2	−0.0352	.0258	−1.3641

Simple Regression

Variable	Y_1	Y_2
Coefficient	1.1456	−.0251
Standard Error	.3690	.0275
T statistic	3.1043	−.9123
Simple r	.3668	−.1151
F statistic	9.6364	.8323

TABLE IV

PER CAPITA JUDICIAL EXPENDITURES

Multiple Regression

Number of Observations 64
Multiple R .... .5670
F Statistic 14.4525

Variable	Coefficient	Standard Error	T Statistic
Y_1	.6415	.1428	4.4918
Y_2	−.0347	.0099	−3.4701

Simple Regression

Variable	Y_1	Y_2
Coefficient	.5822	−.0293
Standard Error	.1539	.0113
T Statistic	3.7835	−2.5821
Simple r	.4331	−.3116
F Statistic	14.3145	6.6670

TABLE V

REMAINING EXPENDITURES PER CAPITA

Multiple Regression

Number of Observations 64
Multiple R .... .5158
F Statistic 11.0561

Variable	Coefficient	Standard Error	T Statistic
Y_1	2.1926	.4784	4.5828
Y_2	−0.0533	.0334	−1.5935

Simple Regression

Variable	Y_1	Y_2
Coefficient	2.1015	.0350
Standard Error	.4809	.0382
T Statistic	4.3702	−.9160
Simple r	.4853	−.1156
F Statistic	19.0986	.8391

pated influences of the institutional phenomena are not fully confirmed in these empirical results. However, these data do suggest that the model has explanatory power. The results do not contradict the predictions of the theory. Granted the unrealistic nature of some of the assumptions and the crudeness of the two propositions, these would seem to be encouraging empirical results.

VII. CONCLUDING COMMENTS

Several interrelated questions merit further attention. First, in a sense this political-economic model of local expenditures represents little more than an effort to relate the preferences of the voters to the decisions on expenditures of democratically selected politicians. As such it may have merit in and of itself; for the formal model stands quite apart from the strict assumptions which were necessary to relate it to the empirical results. In this regard, it is important to point out the conceptual (if not so practicable) possibility of subjecting this theory to an adequate empirical test. One can imagine determining the frequency of preferred points of the registered voters in a locality by survey methods and then relating actual decisions on expenditures to this frequency. Yet, the crude procedure used in this paper also has merit. Granted the appropriateness of the two propositions, by augmenting the model with the additional and unrealistic assumptions in order to relate the theory to the empirical results, one gains insight into the problem of the relative importance of various assumptions. The fact that the results of the regressions do not contradict the theory suggests that the augmented theory has explanatory power and that property holdings are important determinants of expenditure decisions. The fact that the fits are not exceptional tends to indicate that other factors also are important. As a rough attempt to determine what these factors might be, other variables such as median income, median education, etc., were added as independent variables in the regressions; but no significant improvement was made in the proportion of variance explained. At least to the authors, this particular result tends to suggest that differences in preferences for expenditures on the part of the voters may be an important explanatory factor.[21]

It also is important to point out that even within the context of this model several theoretical questions remained unanswered. For example, Theorem 2 depends both upon the presumption that the functions h (the taste for public expenditures) are the same for all voters in a locality and upon the assumption that utility functions are "separable" between private goods and public expenditures so that (12) obtains. Additional work aimed at relaxing both of these assumptions appears important.

Finally, although it is appropriate to base a theory of local governmental expenditure upon the assumption of a property tax, it should be noted that one might just as easily presume that the source of governmental revenues is an income tax and that under such conditions results quite similar to those derived herein follow naturally. However, the authors do not feel that such a step would make this model completely applicable to the allocation of Federal expenditures since the phenomenon of Congressional appropriation, where there are many electorates to be considered instead of one, would be omitted.[22] The model described herein appears more appropriate for situations in which the political process is relatively simple.

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[21] This suspicion is further reinforced by the results concerning the variables of "taste" which are reported in Davis [3].

[22] See Davis, Dempster, and Wildavsky [4, 5] for an empirical study of Congressional appropriation from the "behavioral" point of view.

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APPENDIX

SEPARABILITY AND DOMINANCE

Let $X = (x_1, x_2, \cdots, x_u)$ represent the vector of the u governmental expenditures and $Q_i = (q_{i1}, q_{i2}, \cdots, q_{is})$ be the vector of the i^{th} individual's consumption of the s private goods during the period under consideration. The separable (with respect to public and private goods) utility function of the i^{th} individual is given as

$$(15)\quad f_i(Q_i, X) = g_i(Q_i) + h(X)$$

and it is assumed that both g_i and h are concave (downwards). Note that all voter-consumers share the same utility function h for public expenditures. Let $\delta = \Sigma x_t$ represent the total governmental expenditure under the balanced budget rule defined by (8). Consider the maximization of $h(X)$ for an arbitrarily fixed level of total expenditures δ. Denote by $h(X(\delta))$ the maximum of $h(X)$ for a fixed δ and let the solution be called $X(\delta)$. Then $X(\delta)$ is a solution of the Lagrangian equations:

$$(16)\quad \begin{aligned} &\frac{\partial h}{\partial x_t} - \lambda = 0; \quad t = 1, \cdots, u \\ &\Sigma x_t - \delta = 0 \end{aligned}$$

and $h(X(\delta))$ is a function of δ. It is convenient to express conditions (16) in vector notation. Let $\epsilon = (1, 1, \cdots, 1)'$ be a column vector composed of u unit components, and

$$\Delta h = \left(\frac{\partial h}{\partial x_1}, \frac{\partial h}{\partial x_1}, \cdots, \frac{\partial h}{\partial x_u}\right)$$

represent a row vector of the indicated partial derivatives. Then conditions (16) can be written

$$(17)\quad \begin{aligned} \Delta h(X(\delta)) &= \lambda(\delta)\epsilon' \\ X(\delta)\epsilon &= \delta \end{aligned}$$

where ϵ' is the transpose of ϵ and $\lambda(\delta)$ is written for λ so as to indicate that the value of λ depends upon the value of δ.

Lemma 1: $h(X(\delta))$ is a concave (downwards) function of δ. In other words, as δ varies the maximum value that h assumes is concave.

Proof: To establish concavity (downwards), it is sufficient to show $d^2h/d\delta^2 < 0$. Let

$$X_1 = \left(\frac{dx_1}{d\delta}, \frac{dx_2}{d\delta}, \cdots, \frac{dx_u}{d\delta}\right)'$$

be the column vector of derivatives of the x_t with respect to δ, and also let

$$X_2 = \left(\frac{d^2x_1}{d\delta^2}, \frac{d^2x_2}{d\delta^2}, \cdots, \frac{d^2x_u}{d\delta^2}\right)'$$

represent the column vector of second derivatives of the x_t with respect to δ. Then by the chain rule

$$(18.1) \qquad \frac{dh}{d\delta} = \Delta h X_1$$

$$(18.2) \qquad \frac{d^2h}{d\delta^2} = \Delta h X_2 + \sum_{t,k} \frac{\partial^2 h}{\partial x_t \partial x_k} \frac{dx_t}{d\delta} \frac{dx_k}{d\delta}$$

and attention is centered on (18.2). Differentiating the second of the constraints (17) twice with respect to δ,

$$(19) \qquad X_2'\epsilon = 0$$

so that the vector X_2 is orthogonal to the vector ϵ. Note from the first of the constraints (17) that the vector Δh and the vector ϵ differ only by the scalar $\lambda(\delta)$ so that Δh and ϵ have the same direction. Therefore, Δh is orthogonal to X_2 and

$$(20) \qquad \Delta h X_2 = 0$$

so that all that remains is to examine the second of the terms on the right hand side of (18.2). However, this second term is negative due to the fact that $h(X)$ is concave (downwards) since the negativity of this term is equivalent to presuming that the matrix of coefficients for the second derivatives is negative definite. Hence, $d^2h/d\delta^2 < 0$ and $h(X(\delta))$ is concave. QED.

Let $p = (p_1, \cdots, p_s)$ be the row vector of prices of private goods. For a fixed X, and therefore a fixed δ, consider the maximization problem of the i^{th} voter-consumer:

$$\max g_i(Q_i)$$

$$(21) \qquad \text{subject to}$$

$$pQ_i' + \delta P_i/\Sigma P_j = I_i$$

and let the solution to this problem be denoted $Q_i(\delta)$.

Lemma 2: $g_i(Q_i(\delta))$ is a concave (downwards) function of δ.

Remark: Let $\sigma_i(\delta)$ represent the Lagrangian multiplier of the constraint of (21) for a given δ, and

$$\Delta g_i = \left(\frac{\partial g_i}{\partial q_{i1}}, \cdots, \frac{\partial g_i}{\partial q_{is}}\right)$$

represent a row vector of the indicated partial derivatives. Then the conditions for a solution of (21) are:

$$(22) \qquad \Delta g_i(Q_i(\delta)) = \sigma_i(\delta)p$$

$$pQ_i(\delta)' = I_i - \delta P_i/\Sigma P_j$$

and the proof of Lemma 2 is the same as that for Lemma 1.

Since the sum of concave functions is also a concave function, it follows that

$$(23) \qquad f_i(Q_i(\delta), X(\delta)) = g_i(Q_i(\delta)) + h(X(\delta))$$

is concave (downwards). Obviously, each voter-consumer has a preferred point $\bar{X}_i$ and associated with this preferred point there is an implied total expenditure δ_i. Call these δ_i the desired total expenditures. Note that $\bar{X}_i = X(\delta)_i$. Let the voter-consumers be renumbered in order of ascending values of the δ_i so that $\delta_i \leq \delta_{i+1}$. Denote by δ_m the (not necessarily unique) median of the N numbers δ_i.

Theorem 2: Given the characteristics of the utility functions (15) and the assumed rules of voters' choices between two candidates, then the platform $X(\delta_m)$ is dominant as a political strategy in the sense that no other platform exists which, when opposed by $X(\delta_m)$, gives a larger utility return to at least one-half of the voter-consumers.

Proof: Let the politician Prime select the strategy $X(\delta_m)$ and the politician Star choose the platform X^* where $X^* \neq X(\delta_m)$ and the total expenditure δ^* associated with X^* satisfies the relationship $\delta^* \leq \delta_m$. There are two cases to be examined. In each instance an arbitrary i^{th} voter-consumer is examined for which the relationship of the desired total expenditures $\delta_i \geq \delta_m$ obtains.

Case 1: Assume $\delta^* = \delta_m$. By definition

(15), f_i is separable between private goods and public expenditures so that

$$g_i(Q_i(\delta^*)) = g_i(Q_i(\delta_m)).$$

However, $h(X)$ is concave (downwards), by assumption is the same for all voters, and by construction $h(X(\delta_m)) \geq h(X^*)$ for all $X^* \neq X(\delta_m)$ such that $\delta^* = \delta_m$. Therefore

$$(24) \quad f_i(Q_i(\delta^*), X^*) \leq f_i(Q_i(\delta_m), X(\delta_m))$$

and, by the definition of the median, this relationship obtains for at least one-half of the voter-consumers.

Case 2: Assume $\delta^* < \delta_m$. Note that by construction $h(X^*) \leq h(X(\delta^*))$ if

$$X^* \neq X(\delta^*),$$

so that it may safely be presumed that $X^* = X(\delta^*)$. By appealing to Lemma 1 and Lemma 2, (23) is a concave (downwards) function of δ; and by construction (23) attains its maximum at δ_i. By assumption, $\delta^* < \delta_m \leq \delta_i$. Therefore, it follows that

$$(25) \quad f_i(Q_i(\delta^*), X(\delta^*)) \leq f_i(Q_i(\delta_m), X(\delta_m))$$

so that, by the definition of the median, this relationship obtains for at least one-half of the voter-consumers.

It is obvious that this argument is reflexive so that it holds for all strategies $X^* \neq X(\delta_m)$ such that the associated $\delta^* \geq \delta_m$. QED.

Since Theorem 2 does not depend upon any particular distribution of income and since the g_i are not required to be the same for all voters, there is no particular a priori relationship between the δ_i and the property ratios $P_i/\Sigma P_j$. Quite obviously, however, if it is assumed that both the g_i and the incomes of all voters are identical, and if it is further assumed that none of the public expenditures are inferior goods, then the δ_i and the property ratios $P_i/\Sigma P_j$ have an inverse relationship with one another. In this instance, the median of the desired total expenditures δ_m is identified with the median property ratio $P_m/\Sigma P_j$.